NAG Toolbox for MATLAB

s14ab

1 Purpose

s14ab returns the value of the logarithm of the Gamma function, $\ln \Gamma(x)$, via the function name.

2 Syntax

```
[result, ifail] = s14ab(x)
```

3 Description

s14ab calculates an approximate value for $\ln \Gamma(x)$. It is based on rational Chebyshev expansions.

Denote by $R^i_{n,m}(x) = P^i_n(x)/Q^i_m(x)$ a ratio of polynomials of degree n in the numerator and m in the denominator. Then:

for $0 < x \leq 1/2$,

$$\ln \Gamma(x) \approx -\ln(x) + xR^1_{n,m}(x+1);$$

for $1/2 < x \leq 3/2$,

$$\ln \Gamma(x) \approx (x-1)R^1_{n,m}(x);$$

for $3/2 < x \leq 4$,

$$\ln \Gamma(x) \approx (x-2)R^2_{n,m}(x);$$

for $4 < x \leq 12$,

$$\ln \Gamma(x) \approx R^3_{n,m}(x);$$

and for $12 < x$,

$$\ln \Gamma(x) \approx \left(x - \frac{1}{2}\right)\ln(x) - x + \ln\left(\sqrt{2\pi}\right) + \frac{1}{x}R^4_{n,m}\left(1/x^2\right). \quad (1)$$

For each expansion, the specific values of n and m are selected to be minimal such that the maximum relative error in the expansion is of the order 10^{-d}, where d is the maximum number of decimal digits that can be accurately represented for the particular implementation (see x02be).

Let ϵ denote ***machine precision*** and let x_{huge} denote the largest positive model number (see x02al). For $x < 0.0$ the value $\ln \Gamma(x)$ is not defined; s14ab returns zero and exits with **ifail** $= 1$. It also exits with **ifail** $= 1$ when $x = 0.0$, and in this case the value x_{huge} is returned. For x in the interval $(0.0, \epsilon]$, the function $\ln \Gamma(x) = -\ln(x)$ to machine accuracy.

Now denote by x_{big} the largest allowable argument for $\ln \Gamma(x)$ on the machine. For $\left(x_{\mathrm{big}}\right)^{1/4} < x \leq x_{\mathrm{big}}$ the $R^4_{n,m}\left(1/x^2\right)$ term in Equation (1) is negligible. For $x > x_{\mathrm{big}}$ there is a danger of setting overflow, and so s14ab exits with **ifail** $= 2$ and returns x_{huge}.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J and Hillstrom K E 1967 Chebyshev Approximations for the Natural Logarithm of the Gamma Function *Math.Comp.* **21** 198–203

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $\mathbf{x} > 0.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{x} \leq 0.0$. If $\mathbf{x} < 0.0$ the function is undefined; on soft failure, the function value returned is zero. If $\mathbf{x} = 0.0$ and soft failure is selected, the function value returned is the largest machine number (see x02al).

ifail $= 2$

On entry, $\mathbf{x} > x_{\mathrm{big}}$ (see Section 3). On soft failure, the function value returned is the largest machine number (see x02al).

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively, and E be the absolute error in the result.

If δ is somewhat larger than ***machine precision***, then

$$E \simeq |x \times \Psi(x)|\delta \qquad \text{and} \qquad \epsilon \simeq \left|\frac{x \times \Psi(x)}{\ln \Gamma(x)}\right|\delta$$

where $\Psi(x)$ is the digamma function $\frac{\Gamma'(x)}{\Gamma(x)}$. Figure 1 and Figure 2 show the behaviour of these error amplification factors.

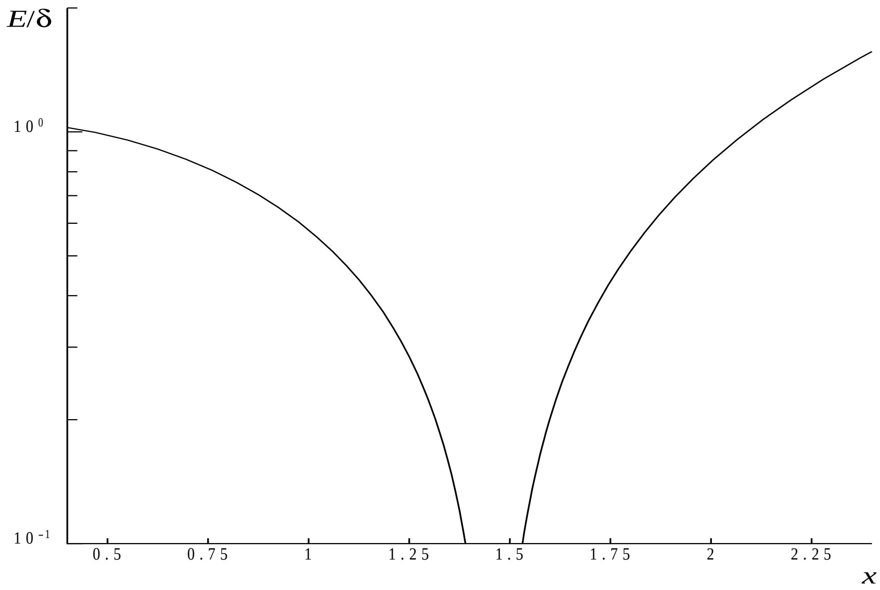

Figure 1

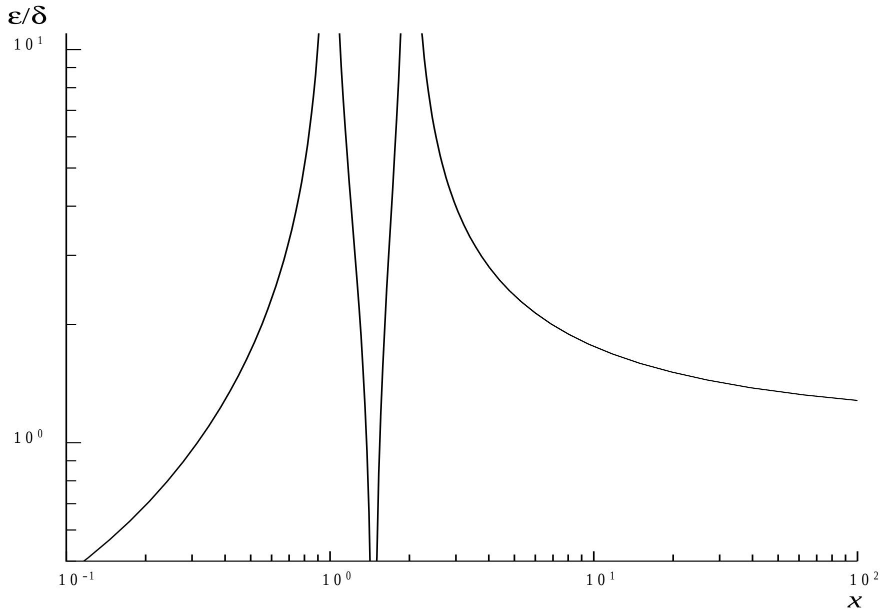

Figure 2

These show that relative error can be controlled, since except near $x = 1$ or 2 relative error is attenuated by the function or at least is not greatly amplified.

For large x, $\epsilon \simeq \left(1 + \frac{1}{\ln x}\right)\delta$ and for small x, $\epsilon \simeq \frac{1}{\ln x}\delta$.

The function $\ln \Gamma(x)$ has zeros at $x = 1$ and 2 and hence relative accuracy is not maintainable near those points. However absolute accuracy can still be provided near those zeros as is shown above.

If however, δ is of the order of ***machine precision***, then rounding errors in the function's internal arithmetic may result in errors which are slightly larger than those predicted by the equalities. It should be noted that even in areas where strong attenuation of errors is predicted the relative precision is bounded by the effective machine precision.

8 Further Comments

None.

9 Example

```
x = 1;
[result, ifail] = s14ab(x)
```

```
result =
     0
ifail =
           0
```